Does the flow of information in a landscape have direction?

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[1] There is an emerging viewpoint that the sediment flux at a given point on the landscape may be influenced by landscape properties in a region extending away from the point of interest. Using a general sediment transport model that incorporates this non-local nature via fractional derivatives, we find a strong asymmetry in the direction in which the non-locality affects a given point: For erosional landscapes, physically plausible elevation profiles are obtained only when the spatial influence is restricted to the region upstream of the point of interest. By contrast, in depositional landscapes, the non-local model is guaranteed to produce physically plausible topographic profiles only when the spatial influence is restricted to the region downstream of the point of interest. These results suggest that information flows downstream in an erosional landscape and upstream in depositional landscapes. **Citation:** Voller, V. R., V. Ganti, C. Paola, and E. Foufoula-Georgiou (2012), Does the flow of information in a landscape have direction?, *Geophys. Res. Lett.*, *39*, L01403, doi:10.1029/2011GL050265.

1. Background

[2] How is geomorphic information at a point in a landscape passed to surrounding points? One obvious means of transfer is via wave-like propagation through the channel network on the landscape surface. In general, the direction of such propagation can be both up- and downstream and is independent of the landscape's uplift/subsidence regime [*Allen*, 2008]. Specific examples include the up- and downstream migration of meanders (depending on the channel aspect ratio and meander wave number) [*Seminara*, 2006; *Zolezzi and Seminara*, 2001; *Zolezzi et al.*, 2005], upstream migration of erosional fronts [*Tucker and Slingerland*, 1994], and upstream-propagating waves of deposition [*Hoyal and Sheets*, 2009]. In contrast, the spatial convolution integrals at the core of the emerging non-local models of landscape dynamics [*Schumer et al.*, 2009; *Foufoula-Georgiou et al.*, 2010; *Voller and Paola*, 2010] imply that conditions at a point may depend intrinsically on conditions elsewhere in the landscape, without the need for a wave-like propagation. Here, by considering a simple realization of a source-to-sink sediment transport model, we show that, in this non-local framework, physically plausible solutions for fluvial long profiles require a binary partitioning in the direction of influence between erosional and depositional landscapes. In erosional landscapes the non-locality is directed upstream, i.e., sediment flux at a point depends on information from features of the landscape *upstream* of the point in question. Depositional systems present the mirror image, where the non-locality is directed downstream and the sediment flux is controlled by information from the *downstream* landscape features. In the non-local framework of fractional diffusion, the direction in which information is passed and recorded in landscapes is fundamentally dependent on their mean state of mass balance (net loss versus net gain).

2. A Basic Long-Profile Transport Model

[3] The starting point in the analysis is a model of the mean fluvial surface topography along the flow path from source to sink (the "long profile"). A basic realization of this system, which takes account of the major sediment production and deposition in the system, comprises a contiguous domain of an erosional uplifting region connected to a subsiding depositional basin. In this way, a local balance between erosion (deposition) and uplift (subsidence) results in a steady-state laterally averaged fluvial profile measured by the elevation profile $h(x)$. The governing equation is the steady Exner equation [*Paola and Voller*, 2005]. With suitable scaling and the assumptions (without loss of generality for the current analysis) that the erosional and depositional domains equally divide the domain and undergo piston uplift/subsidence (+1/−1), the steady Exner leads to two separate problem statements, one for the erosional region

$$\frac{dq}{dx} = 1, \quad 0 \le x \le \frac{1}{2}; \quad q(0) = 0, \quad h\left(\frac{1}{2}\right) = 0 \tag{1}$$

and the other for the depositional region

$$\frac{dq}{dx} = -1, \quad \frac{1}{2} \le x \le 1; \quad h\left(\frac{1}{2}\right) = 0, \quad q(1) = 0 \tag{2}$$

where $q(x)$ is the unit sediment discharge and a reference elevation $h(\frac{1}{2}) = 0$ is imposed. In systems where the channels on the fluvial surface can set their own width, reasonable first order approximations of sediment transport theory suggest that the unit discharge is proportional to the local fluvial slope [*Paola et al.*, 1992]—a linear diffusion law—i.e.,

$$q = -\frac{dh}{dx} \tag{3}$$

This definition can be used in (1) and (2) to find the steady state fluvial profile

$$h = \frac{\left(\frac{1}{2}\right)^2 - x^2}{2}; \qquad 0 \le x \le \frac{1}{2} \tag{4a}$$

$$h = \frac{\left(\frac{1}{2}\right)^2 - (1-x)^2}{2}; \quad \frac{1}{2} \le x \le 1 \tag{4b}$$

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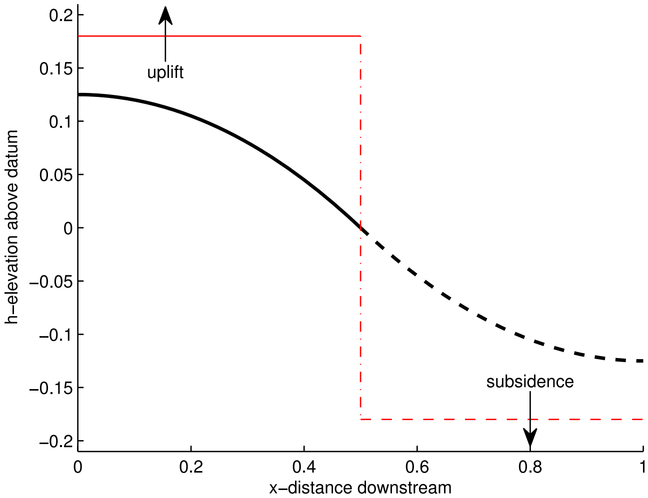

Figure 1. Fluvial profile predicted with local Exner model (equations (4a) and (4b)). The solid line is thy fluvial profile in the uplift/erosional domain, the broken line is the profile in the subsiding/depositional domain. Note the key physical attributes of a concaved-down erosional profile with maximum elevation at the upstream domain boundary (x = 0) and a concave-up depositional profile with minimum elevation at the domain boundary (x = 1).

A plot of the elevation from (4) (Figure 1) exhibits—subject to the assumption of linear diffusion transport—a fluvial long profile consisting of a concave-down erosional profile with maximum elevation at the upstream domain boundary (x = 0) and a concave-up depositional profile with a minimum elevation at the downstream boundary (x = 1).

3. A Non-local Model

[4] A key feature in the model presented above is a flux given in term of a local quantity; the fluvial slope. The underlying assumption of any local model is a separation of scales between the scales of heterogeneity of the system and the scales of transport. However, landscapes are shaped by processes that have heavy-tailed distribution (e.g., streamflows, precipitation, etc.) and the landscapes themselves are known to exhibit variability over a range of scales (that is manifested in the fractal structure of landscapes [*Trucotte*, 1992]). These observations have driven an emerging view that transport in landscapes may be better treated with non-local models [e.g., *Schumer et al.*, 2009; *Foufoula-Georgiou et al.*, 2010; *Voller and Paola*, 2010]. Within this approach, diffusive flux is estimated as a power-law weighted sum of slopes across a region that could extend both up- and downstream of the point of calculation. Recent results have shown that such diffusive models can provide a better match to the observed convexities of erosional hill-slopes [*Foufoula-Georgiou et al.*, 2010] and experimental depositional fluvial profiles [*Voller and Paola*, 2010]. In this light, we explore the consequences of introducing a non-local treatment in our example erosion-depositional diffusive landscape transport model.

[5] A standard approach for formally introducing non-locality into the Exner transport model—appropriate when the heterogeneity length scales on the fluvial surface are power-law distributed—is to use fractional calculus [*Podlubny*, 1998]. In this way, recalling that in our simple model the erosional and depositional components form a single contiguous domain, a non-local expression for the unit discharge can be written as a weighted average of the left- and right- handed Caputo derivatives, i.e.,

$$q(x) = \frac{1+\beta}{2}\left[\frac{(-1)}{\Gamma(1-\alpha)}\int_0^x (x-\xi)^{-\alpha}\frac{dh(\xi)}{d\xi}\,d\xi\right] + \frac{1-\beta}{2}\left[\frac{1}{\Gamma(1-\alpha)}\int_x^1 (\xi-x)^{-\alpha}\frac{dh(\xi)}{d\xi}\,d\xi\right] \quad (5)$$

or in more compact notation

$$q(x) = \frac{1+\beta}{2}\left[-\frac{d^\alpha h}{dx^\alpha}\right] + \frac{1-\beta}{2}\left[\frac{d^\alpha h}{d(-x)^\alpha}\right] \quad (6)$$

In these expressions the parameter α, $0 < \alpha \leq 1$, is a measure of the locality. A value of $\alpha = 1$ indicates pure locality: the unit discharge is determined only by the local fluvial slope $-dh/dx$. On the other hand, as $\alpha \to 0$ the value of the unit discharge depends equally on all the slopes throughout the integration domain and is thus entirely non-local. In addition to the measure of locality it is also important to recognize the difference between the two components on the right-hand side of (5). In the first component the integration is over the region upstream of the point x, indicating that only upstream features in the landscape control the non-locality. By contrast, in the second component the integration is over the downstream region, thereby indicating that the non-locality is controlled by downstream features and conditions. Thus the parameter β, $-1 \leq \beta \leq 1$ in (5) is a directional measure, providing a weighting between purely upstream (β = 1) and purely downstream ($\beta = -1$) non-locality.

[6] It is worth noting at this point that there are a number of alternative definitions for a fractional derivative—see *Podlubny* [1998] for a general treatment of the alternatives or *Voller and Paola* [2010, Appendix] for a discussion in a geophysical setting. Here, we choose the Caputo derivative for two reasons: (i) it is not singular at the origin and (ii) Caputo derivatives of a constant are zero. These features allow us to readily develop closed solutions for our test problem.

4. The Influence of the Direction of Non-local Information

[7] Up to this point, research on the application of equation (5), or its equivalent (6), in modeling landscapes has examined the role of the locality factor α, with a focus on understanding how this value is related to the statistics of physically observed features in the landscape [*Schumer et al.*, 2009; *Foufoula-Georgiou et al.*, 2010; *Voller and Paola*, 2010]. Here we study instead the non- locality direction coefficient $-1 \leq \beta \leq 1$. In particular, on using the basic properties of Caputo derivatives [*Foufoula-Georgiou et al.*, 2010; *Podlubny*, 1998]—that the fractional derivative of a positive power is

$$\frac{d^\alpha}{dx^\alpha}x^\eta \equiv \left[\frac{1}{\Gamma(1-\alpha)}\int_0^x (x-\xi)^{-\alpha}\frac{d\xi^\eta}{d\xi}\,d\xi\right] = \frac{\Gamma(\eta+1)}{\Gamma(\eta+1-\alpha)}x^{\eta-\alpha}; \quad 0 \leq \alpha < 1,\ \eta > 0, \quad (7)$$

Table 1. Solutions of Limit Case Non-local Fluvial Profiles for Erosional and Depositional Systems

Contribution to Non-Locality	Erosional Landscape $0 \le x \le \frac{1}{2}$ Equation (6) in Equation (1)	Depositional Landscape $\frac{1}{2} \le x \le 1$ Equation (6) in Equation (2)
$\beta = -1$ purely downstream	$h = -\frac{(1-x)^{1+\alpha} - \left(\frac{1}{2}\right)^{1+\alpha}}{\Gamma(2+\alpha)} + \frac{(1-x)^{\alpha} - \left(\frac{1}{2}\right)^{\alpha}}{\Gamma(1+\alpha)}$	$h = \frac{(1-x)^{1+\alpha} - \left(\frac{1}{2}\right)^{1+\alpha}}{\Gamma(2+\alpha)}$
$\beta = 1$ purely upstream	$h = -\frac{x^{1+\alpha} - \left(\frac{1}{2}\right)}{\Gamma(2+\alpha)}$	$h = \frac{x^{1+\alpha} - \left(\frac{1}{2}\right)^{1+\alpha}}{\Gamma(2+\alpha)} - \frac{x^{\alpha} - \left(\frac{1}{2}\right)^{\alpha}}{\Gamma(1+\alpha)}$

the derivative of a constant is identically zero, and in the interval $0 \le x \le 1$, the right and left derivatives are related through [*Foufoula-Georgiou et al.*, 2010]

$$\frac{d^{\alpha}}{d(-x)^{\alpha}} \equiv \frac{d}{d(1-x)^{\alpha}}, \quad 0 \le x \le 1 \tag{8}$$

—we can re-solve (1) and (2) with the unit discharge definition in (6) and arrive at non-local expressions for the fluvial profile in the limit cases of $\beta = 1$ or $\beta = -1$ (Table 1). Plots of the source-sink profiles in Table 1 (Figure 2) for a non-locality of $\alpha = 0.7$ show that with a full upstream bias in the locality, the maximum surface elevation, as physically expected, is located at the origin. On the other hand, if we assume a full downstream bias the erosional section of the profile is non-physical, exhibiting a maximum elevation downstream of the origin. In fact, assuming that the profiles for general values of the direction parameter $-1 < \beta < 1$ must be enveloped by these limit solutions, we argue that a physical meaningful solution for the erosion profile, with the maximum elevation located at the origin, can occur only with a full upstream bias $\beta = 1$. Similar arguments lead to the conclusion that a physically meaningful depositional profile solution can occur only with a full downstream bias $\beta = -1$. Hence, to the extent that the fractional calculus provides a valid non-local treatment, the mathematical analysis of long-profiles solutions compared to the physically expected behavior leads to an interesting hypothesis:

[8] Non-locality in an erosional landscape is purely determined by upstream features and conditions, whereas in a depositional landscape the non-locality is controlled by downstream features and conditions.

[9] This hypothesis implies that the flow of information in landscape dynamics depends on the nature of the system considered; in an erosional system, information flows forward, in the downstream direction whereas in a depositional system, information flows backwards, in the upstream direction.

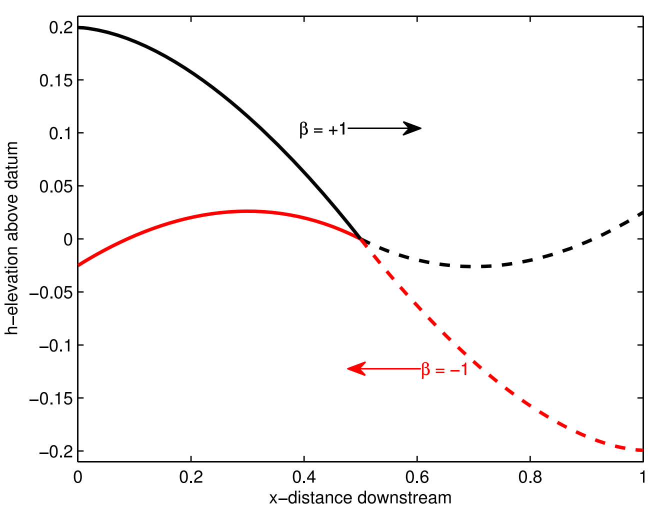

Figure 2. Fluvial profiles predicted with a non-local Exner model ($\alpha = 0.7$ in Table 1). The solid lines are the erosional profiles and the broken lines the depositional profiles. The colors distinguish between the limit solutions; when the non-locality information comes purely from upstream ($\beta = 1$) the line is black, when the non-locality is purely from the downstream ($\beta = -1$) the line is red. Since these limits, envelope all possible solutions, it is readily observed that a physically meaningful solution can only be obtained if the non-locality in the erosional domain is restricted to upstream points and the non-locality in the depositional domain is restricted to downstream points. Any solution obtained away from these limits would result in a non-physical elevation maximum and/or minimum at internal points of the domain.

5. Discussion

[10] What might explain the observed switch in non-locality direction between the erosional and depositional cases? We turn first to the mathematical nature of the problem. Within the mathematical construct, the feature that ultimately controls the locations of the extrema in the domain is the zero unit discharge boundary conditions, located at $x = 0$ in the erosional system and at $x = 1$ in the depositional system. In local models this corresponds to specifying a zero first derivative in the fluvial local slope, $dh/dx = 0$, at these locations. In the problems of interest, a zero local slope at the appropriate domain boundary is sufficient to ensure that a non-physical maximum or minimum does not occur within the domain $0 < x < 1$. In the erosional system, by taking the appropriate integer derivative of the solutions in Table 1, it can be seen that when the non-locality is fully biased upstream ($\beta = 1$) the local slope at $x = 0$ is identically zero for all choices of $0 < \alpha \le 1$whereas the slope when the non-locality is fully biased downstream ($\beta - 1$) is strictly positive for all values of $\alpha < 1$ (see right hand column in Table 2). On accepting that the predictions for fluvial profiles obtained with non-extreme values of non-locality direction, i.e., $-1 < \beta < 1$, will be enveloped by the solutions at the extremes (shown in Figure 2 and Tables 1 and 2) it follows that, for any choice where $\beta \ne 1$ and $\alpha \ne 1$, the local gradient dh/dx at $x = 0$ will be strictly positive. In this circumstance the resulting fluvial surface has to exhibit a (non-physical) maximum downstream of $x = 0$. This behavior is mirrored in the depositional system, as indicated in the right hand column in Table 2. When the non-locality is fully biased down-stream ($\beta = -1$) the local gradient at $x = 1$ is identically zero for all choices $0 < \alpha \le 1$ leading to

Table 2. Physical Slopes (dh/dx) at End Points for Solutions in Table 1.

Contribution to Non-locality	Erosional Landscape $0 \le x \le \frac{1}{2}$	Depositional Landscape $\frac{1}{2} \le x \le 1$
$\beta = -1$ purely downstream	$\frac{dh}{dx}\Big\rvert_{x=0} = \frac{1-\alpha}{\Gamma(1+\alpha)}$	$\frac{dh}{dx}\Big\rvert_{x=1} = 0$
$\beta = 1$ purely upstream	$\frac{dh}{dx}\Big\rvert_{x=0} = 0$	$\frac{dh}{dx}\Big\rvert_{x=1} = \frac{1-\alpha}{\Gamma(1+\alpha)}$

prediction of a fluvial surface that does not exhibit a (non-physical) minimum at an internal domain point. Once any upstream non-locality ($\beta \neq -1$ and $\alpha \neq 1$) is introduced, however, the local topographic gradient at $x = 1$ is strictly positive requiring the occurrence of a non-physical upstream topographic minimum. This result can be further understood by considering the case where the non-locality is directed fully upstream $\beta = 1$. In this case, through the definition of the Caputo derivative in equation (5), the flux at any point is essentially constructed from a weighted sum of up-stream slopes. As such, if we were to approach the downstream boundary at $x = 1$ in a physically correct manner, with the negative fluvial slope monotonically increasing to zero, we would not be able to meet the flux boundary condition $q(1) = 0$. In fact, under the assumption $\beta = 1$, the only way to meet this condition is to allow for the unphysical appearance of positive slopes as we approach the downstream boundary at $x = 1$.

[11] Physically, the results indicate a reversal in the direction of morphodynamic information flow in erosional versus depositional systems. The most obvious physical asymmetry between erosional and depositional systems is that in the former, channel networks are typically convergent while in the latter they are generally divergent. Perhaps the bias is linked to this, although the erosion-convergence and deposition-divergence correspondence is far from exact in nature. A stricter reading of the results is that, to the extent that sediment transport really is non-local at landscape scales, and that the non-locality is power-law distributed, the limiting non-local process in erosional landscapes is sediment delivery *to* the point in question, while the limiting non-local process in depositional landscapes is removal of material *from* the point in question. One might say that erosional systems are import-controlled while depositional systems are export-controlled.

6. Conclusion

[12] The central contribution of this work is to show that a direct consequence of the emerging non-local models of landscape dynamics based on the fractional calculus [*Schumer et al.*, 2009; *Foufoula-Georgiou et al.*, 2010; *Voller and Paola*, 2010] is a strict directional dependence that is reversed between erosional and depositional systems. As such, experimental or field measurement of the difference in directional control between depositional and erosional systems would be a decisive step toward confirming the physical validity of current non-local sediment transport models. In this respect preliminary analyses are generally positive. Modeling of erosional systems with fully biased up-stream non-locality explains many observed features in erosional hillslope profiles [*Foufoula-Georgiou et al.*, 2010]. Likewise a fully biased downstream model has been shown to explain anomalies in fluvial profiles formed in experimental depositional basins [*Voller and Paola*, 2010].

[13] It is important to recognize, however, that the results obtained here are for a steady-state equilibrium landscape. In landscapes undergoing a transient change—e.g., following a change in a forcing variable such as base-level—the long waiting times in the sediment motions may lead to time fractional derivatives in the governing transport equation [*Schumer et al.*, 2009]. Experimental evidence for the existence of truncated heavy-tailed waiting times is reported in depositional systems under steady-state conditions [*Ganti et al.*, 2011]. However, this non-locality in time will not affect the results presented here as the steady-state solutions remain the same for both time-fractional and non time-fractional cases; the difference being in how fast the steady-state is reached.

[14] In closing we note that many fundamental problems in landscape science can be posed as grand challenge inverse problems, in which one is attempting to interpret landscape-forming dynamics and initial and forcing conditions from spatial and temporal field data, either in extant landscapes or sedimentary records. Inverse problems are inherently ill conditioned and an understanding of data and information flows is thus vital to both interpretation and understanding error propagation. In this context, recognizing the potential for a mirror difference in direction of the flow of information between an erosional and depositional system is important for any inverse geological analysis.

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